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Efficient solvers for soft-constrained MPC

Gianluca Frison, John Bagterp Jørgensen

Abstract— **The ability of easily and naturally handling constraints is certainly one of the winning features of Model Predictive Control (MPC). The use of hard output constraints, however, is often not physically necessary, and furthermore it can lead to unfeasible optimization problems. One way to avoid this issue is the use of soft-constraints on the outputs (and more in generals on the states). In the soft-constrained formulation, the constraint may be violated, but incurring in a penalty cost: the optimization procedure thus avoid the violation of these constraints whenever possible. Soft-constraints are traditionally handled by introducing a decision variable for each slack variable associated with the soft-constraints. This increases the size of the dynamic system variables, and therefore the size of the optimization problem, and it increases remarkably the solution time. In this paper, we want to show that IP and ADMM methods for box-constrained MPC can be modified to handle the case of soft-constraints on the states, and at a similar cost-per-iteration. This is obtained by exploit the special structure of the KKT system of the soft-constrained MPC problem, avoiding the introduction of additional control variables. As a consequence, each iteration of the IP or ADMM methods requires the solution of an unconstrained MPC sub-problem with the same size as in the case of box-constrained MPC.**

I. Introduction

Model Predictive Control (MPC) is probably the most successful advance control technique in industry [6]. It makes use of a plant model to predict the future evolution of the plant dynamic and compute an input sequence optimal with respect to some cost function. At each sampling instant, only the first input of this optimal sequence is applied to the plant, before a new input sequence is computed using the latest measurements: thus, at each sampling instant an optimization problem has to be solved in real-time. This has traditionally limited the use of MPC to system with slow dynamic, as in process or chemical industry. In recent years MPC has been successfully applied to system with fast dynamic, with sampling times also in the micro-seconds range [4]: these improvements are due to both faster hardware as well as the use of structure-exploiting algorithms.

One of the winning features of MPC is certainly its ability of easily and naturally handling constraints [5]. However, the presence of constraints makes computationally-expensive the solution of optimization problems. Therefore, algorithms exploiting special constraints formulations (e.g. box constraints) have been proposed [1], [8]. One drawback of the use of hard-constraints is that they may make the optimization problem unfeasible: this is especially true in the case of output constraints. Furthermore, often the use of hard-constraints is not physically necessary.

One way to avoid this issue is the use of soft-constraints on the outputs (and more in general on the states). In this formulation, the constraint may be violated, but incurring on a penalty cost. This is usually obtained by introducing slack variables associated with the soft constrained, and heavily penalizing them: the optimization algorithm keeps these slack variables to zero whenever possible, and violates the constraints only if necessary. Soft-constraints are usually handled by introducing a decision variable for each slack variable associated with the soft-constraints. This approach has the advantage of formulating the optimization problem in the form of an hard-constrained one. However, this comes at a cost from a computational point of view: the simple constraint structure is lost (and thus algorithms for general constraints must be employed), and furthermore the extra decision variables enter in the optimization problem as dynamic system variables, that typically contribute with a cubic term in the flop count. Recently, a different formulation has been proposed [7], avoiding the introduction of extra optimization variables: however, this comes at the cost of approximating of the soft constraint penalty

In this paper, we propose a different approach. We want to show that both IP and ADMM methods for box-constrained MPC can be modified to handle the case of soft-constraints on the states, and that the flop count increases only by a linear term. This is obtained by exploit the special structure of the KKT system associated with the soft-constrained MPC problem: new optimization variables are introduced for the slack variables, but these are not additional control variables. As a consequence, each iteration of the IP and ADMM methods requires the solution of an unconstrained MPC sub-problem (accounting for cubic and quadratic terms in the flop count) with the exact same structure and size as in the case of box-constrained MPC, and that can be solved efficiently [2], [3].

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